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IDENTIFICATION OF THE CONSTANTS OF HEAT TRANSFER IN ARBITRARY THERMAL MODES USING LOCAL HEAT SOURCES

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Exact explicit relations connecting thermophysical characteristics of materials with results of measuring nonstationary values of primary parameters in heating of specimens by local heat sources with power varying arbitrarily in time are presented.

In thermophysical experiments, heating of specimens of investigated materials by local heat sources of various configurations is employed extensively in practice [1-3]. It should be noted that calculational relations for thermophysical characteristics (TPC) are approximate in most cases and, moreover, are derived for specific laws of the time variation of the supplied heat fluxes. The current study, in developing the approach delineated in [4], presents a solution to the problem of determining a TPC complex using results of measuring nonstationary temperatures and heat fluxes in heating of specimens by surface local heat sources of variable power.

As applied to a two-dimensional process of heat propagation in a material, we consider the mathematical model

$$r^{-k}\frac{\partial}{\partial r}r^{k}\frac{\partial T(r,z,\tau)}{\partial r}+\frac{\partial^{2}T(r,z,\tau)}{\partial z^{2}}=\frac{1}{a}\frac{\partial T(r,z,\tau)}{\partial \tau}, \tag{1}$$

$$-\lambda\frac{\partial T(r,z,\tau)}{\partial z}\Big|_{z=0}=q(r,\tau), \tag{2}$$

$$T(r,z,\tau)\Big|_{r\to\infty}=T(r,z,\tau)\Big|_{z\to\infty}=0, \tag{2'}$$

$$T(r,z,0)=0, \tag{2''}$$

which holds, for example, for heating of a half-space by surface local heat sources forming temperature fields symmetric about a vertical axis z ($k = 1$) or about a vertical plane $x0z$ ($k = 0$).

Using the Laplace transform with respect to the variable τ and an integral transformation with respects to the coordinate r:

$$T(p,z,s)=\int_{0}^{\infty}p^{\frac{1-k}{2}}r^{\frac{1+k}{2}}J_{\frac{k-1}{2}}(pr)\,T(r,z,s)\,dr,$$

$$T(r,z,s)=\int_{0}^{\infty}p^{\frac{1+k}{2}}r^{\frac{1-k}{2}}J_{\frac{k-1}{2}}(pr)\,T(p,z,s)\,dp,$$

where $J_{\frac{k-1}{2}}$ is the Bessel function of order $\frac{k-1}{2}$, we write the solution to problem (1)-(2'') as follows:

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$$T(r, z, s) = \frac{1}{\lambda} \int_0^\infty \frac{p^{\frac{1+k}{2}} r^{\frac{1-k}{2}} J_{\frac{k-1}{2}}(pr)\, q(p, s)\, dp}{(p^2 + s/a)^{1/2}}, \tag{3}$$

where $q(p, s) = \int_0^\infty p^{\frac{1+h}{2}} \cdot r^{\frac{1-h}{2}} \cdot J_{\frac{k-1}{2}}(pr) q(r, s) dr$, and $q(r, s)$ is the heat flux from the source to the surface.

With reference to the sources concentrated on a circle of radius R_0 or distributed uniformly within a circle of radius R_0, at $r = 0$ and $z = 0$ we have the following expressions for the temperature:

$$T(0, 0, s) = \frac{Q(s)}{2\pi R_0 \lambda} \exp\left(-\sqrt{\left(\frac{s}{a}\right)} R_0\right), \tag{4}$$

$$T(0, 0, s) = \frac{Q(s)}{\pi R_0^2 \sqrt{\lambda c\rho}\, \sqrt{s}} \left[1 - \exp\left(-\sqrt{\left(\frac{s}{a}\right)} R_0\right)\right], \tag{5}$$

where $Q(s)$ in the total power of a source. It should be pointed out that, for a source concentrated at a point, relation (4) is valid at the distance R_0 from the source.

For sources concentrated on an infinite line, at the distance R_0 from the source we have

$$T(R_0, 0, s) = \frac{Q^*(s)}{\pi\lambda} K_0\left(\sqrt{\left(\frac{s}{a}\right)} R_0\right); \tag{6}$$

and for a source distributed uniformly in a band of width $2R_0$, at the source center we have

$$T(0, 0, s) = \frac{Q^*(s)}{2\lambda}\left[K_0\left(\sqrt{\left(\frac{s}{a}\right)} R_0\right) L_{-1}\left(\sqrt{\left(\frac{s}{a}\right)} R_0\right) + \right.$$

$$\left. + K_1\left(\sqrt{\left(\frac{s}{a}\right)} R_0\right) L_0\left(\sqrt{\left(\frac{s}{a}\right)} R_0\right)\right] \tag{7}$$

(Q^* is the source power per unit length; and K_ν and L_ν are the Macdonald and Struve functions, respectively).

Using differentiation with respect to the parameter of the Laplace transform s, it is a simple matter to establish the following relationships, in the transform space, between the TPC and the primary parameters for the sources:

$$T'(s) Q(s) - T(s) Q'(s) = -\frac{R_0}{2\sqrt{a}} s^{-1/2} T(s) Q(s), \tag{8}$$

concentrated on a circle (or at a point);

$$\frac{\sqrt{a}}{R_0} [T'(s) \tilde{q}(s) - T(s) \tilde{q}'(s)] = (\lambda c\rho)^{-1/2} s^{-3/2} q^2(s) - s^{-1} T(s) q(s), \tag{9}$$

distributed within a circle of radius R_0;

$$q(s) = Q(s)/(\pi R_0^2), \quad \tilde{q}(s) = q(s)/\sqrt{s},$$

$$s\left\{[T'(s) Q^*(s) - T(s) Q^{*\prime}(s)]' Q^*(s) - 2Q^{*\prime}(s) [T'(s) Q^*(s) - T(s) Q^{*\prime}(s)]\right\} +$$

$$+ Q^* (s) [T' (s) Q^* (s) - T (s) Q^{*'} (s)] = \frac{R_0^2}{4a} T (s) Q^{*^2} (s) , \tag{10}$$

concentrated on a line;

$$[f' (s) Q^* (s) - 2f (s) Q^{*'} (s)]' Q^* (s) - 3Q^{*'} (s) [f' (s) Q^* (s) - 2f (s) Q^{*'} (s) +$$

$$+ \frac{7}{2} s^{-1} Q^* (s) [f' (s) Q^* (s) - 2Q^{*'} (s) f (s) + 3/2 s^{-2} f (s) Q^{*^2} (s) =$$

$$= \frac{R_0^2}{4a} \left[s^{-1} Q^{*^2} (s) f (s) + \frac{1}{2} T (s) s^{-2} Q^{*^3} (s) \right] , \quad f (s) = T' (s) Q^* (s) - T (s) Q^{*'} (s) , \tag{11}$$

distributed in a band.

Utilizing well-known inversion theorems, from the above relationships we may readily obtain the corresponding functional relations between the TPC and the primary parameters (T, Q) in the space of inverse transforms for the source:

concentrated on a circle (or at a point):

$$a = \frac{R_0^2}{4} \frac{F_1^2 (\tau)}{F_2^2 (\tau)} , \tag{12}$$

where

$$F_1 (\tau) = \pi^{-1/2} \int_0^\tau T (\tau - \theta) \int_0^\theta (\theta - \vartheta)^{-1/2} Q (\vartheta) \, d\vartheta \, d\theta ;$$

$$F_2 (\tau) = \int_0^\tau (\tau - 2\theta) T (\tau - \theta) Q (\theta) \, d\theta ;$$

distributed within a circle

$$\frac{\sqrt{a}}{R_0} \varphi_1 (\tau) + (\lambda c \rho)^{-1/2} \varphi_2 (\tau) = \varphi_3 (\tau) , \tag{13}$$

where

$$\varphi_1 (\tau) = \int_0^\tau (\tau - 2\theta) T (\tau - \theta) \tilde{q} (\theta) \, d\theta ;$$

$$\varphi_2 (\tau) = \int_0^\tau \pi^{-1/2} q (\tau - \theta) \int_0^\theta q (\vartheta) (\theta - \vartheta)^{1/2} \, d\vartheta \, d\theta ;$$

$$\varphi_3 (\tau) = \frac{1}{2} \int_0^\tau T (\tau - \theta) \int_0^\theta q (\vartheta) \, d\vartheta \, d\theta ; \quad \tilde{q} (\theta) = \pi^{-1/2} \int_0^\theta q (\vartheta) (\theta - \vartheta)^{-1/2} \, d\vartheta ;$$

concentrated on a line

$$a = \frac{R_0^2}{4} \frac{\psi_1 (\tau)}{\psi_2 (\tau)} , \tag{14}$$

where

$$\psi_1(\tau) = \int_0^\tau T(\tau - \theta) \int_0^\theta Q^*(\theta - \vartheta) Q^*(\vartheta)\, d\vartheta\, d\theta ;$$

$$\psi_2(\tau) = \int_0^\tau (3\theta - \tau) Q^{*'}(\theta) f(\tau - \theta)\, d\theta + 3 \int_0^\tau f(\tau - \theta) Q^*(\theta)\, d\theta - \tau f(\tau) Q^*(0) ;$$

$$f(\theta) = \int_0^\theta (2\vartheta - \theta) T(\theta - \vartheta) Q^*(\vartheta)\, d\vartheta ;$$

distributed in a band:

$$a = \frac{R_0^2}{4} \frac{\overline{\psi}_1(\tau)}{\overline{\psi}_2(\tau)}, \tag{15}$$

where

$$\overline{\psi}_1(\tau) = \int_0^\tau f(\tau - \theta) \int_0^\theta Q^*(\theta - \vartheta) \widetilde{Q}^*(\vartheta)\, d\vartheta\, d\theta +$$

$$+ \frac{1}{2} \int_0^\tau T(\tau - \theta) \int_0^\theta Q^*(\theta - \vartheta) \widetilde{\widetilde{Q}}^*(\vartheta)\, d\vartheta\, d\theta ;$$

$$f(\theta) = \int_0^\theta (2\vartheta - \theta) T(\theta - \vartheta) Q^*(\vartheta)\, d\vartheta ; \quad \widetilde{Q}^*(\theta) = \int_0^\theta Q^*(\vartheta)\, d\vartheta ,$$

$$\widetilde{\widetilde{Q}}^*(\theta) = \int_0^\theta Q^*(\theta - \vartheta) Q^*(\vartheta)\, d\vartheta ; \quad \overline{\psi}_2(\tau) = \int_0^\tau (3\tau - 4\theta) Q^*(\tau - \theta) \widetilde{f}(\theta)\, d\theta +$$

$$+ \frac{7}{2} \int_0^\tau \widetilde{f}(\tau) \widetilde{Q}^*(\tau - \theta)\, d\theta + 3/2 \int_0^\tau \widetilde{f}(\theta) \widetilde{\widetilde{Q}}^*(\tau - \theta)\, d\theta ;$$

$$\widetilde{f}(\theta) = \int_0^\theta (3\vartheta - \theta) f(\theta - \vartheta) Q^*(\vartheta)\, d\vartheta .$$

The above dependences for the examined types of sources establish exact explicit functional relations between the TPC and the primary parameters (Q, T) for arbitrary laws of the time variation of the source power and an arbitary duration of realization. It is noteworthy that, depending on the source type, the presented relations connect either one constant (thermal diffusivity) or a complex of constants a and $\lambda c\rho$ with the primary parameters. In the first case, for identifying the entire complex of constants we may, apparently, use (having determined the parameter a) relations (4) and (6)-(8) with account for well-known inversion theorems. In the case of sources distributed within a circle, relation (13), with allowance for its invariance relative to realization duration and laws of time variation of the power, permits a determination of each of the parameters a and $\lambda c\rho$ by calculating the functionals φ_1, φ_2, and φ_3 for various time intervals of a single test or for tests with different laws of time variation of the heating power. Evaluations of the sought parameters thus obtained will probably be dependent.

Independent evaluations of the complex of sought parameters can be deduced using the measurement results for realizations with different dimensions of the sources. For example, for sources concentrated on circles of radii R_0 and $2R_0$, based on relation (4) we write

$$T_1(s) = \frac{Q_1(s)}{2\pi R_0 \lambda} \exp\left(-\sqrt{\left(\frac{s}{a}\right)} R_0\right),$$

$$T_2(s) = \frac{Q_2(s)}{4\pi R_0 \lambda} \exp\left(-2\sqrt{\left(\frac{s}{a}\right)} R_0\right),$$

whence

$$\lambda T_1^2(s) Q_2(s) = \frac{1}{\pi R_0} T_2(s) Q_1^2(s),$$

where T_1 and T_2 are the temperatures at the center of the heating zone in the realizations with sources concentrated on circles of radii R_0 and $2R_0$ (or the temperatures at the distances R_0 and $2R_0$ from a point source). In the space of inverse transforms, obviously,

$$\lambda = \frac{\bar{\Psi}_1(\tau)}{\bar{\Psi}_2(\tau)} \tag{16}$$

$$\left(\begin{aligned} \bar{\Psi}_1(\tau) &= \frac{1}{\pi R_0} \int_0^\tau Q_1(\tau - \theta) \int_0^\theta T_2(\theta - \vartheta) Q_1(\vartheta)\, d\vartheta\, d\theta; \\ \bar{\Psi}_2(\tau) &= \int_0^\tau Q_2(\tau - \theta) \int_0^\theta T_1(\theta - \vartheta) T_1(\vartheta)\, d\vartheta\, d\theta \end{aligned}\right).$$

The parameter a can be found in this case from relation (12) using the measurement data for one or another realization. Furthermore, the parameter a can be estimated based on the relationship

$$\frac{T_1(s) Q_2(s)}{T_2(s) Q_1(s)} = \exp\left(-\sqrt{\left(\frac{s}{a}\right)} R_0\right),$$

whence, after differentiation with respect to the parameter s and conversion to the inverse transforms, we have

$$a = \frac{R_0^2 \bar{F}_1^2(\tau)}{4 \bar{F}_2^2(\tau)}, \tag{17}$$

where

$$\bar{F}_1(\tau) = \pi^{-1/2} \int_0^\tau \varphi_1(\tau - \theta) \int_0^\theta \varphi_2(\vartheta)(\theta - \vartheta)^{-1/2}\, d\vartheta\, d\theta, \quad \varphi_1(\theta) = \int_0^\theta T_1(\theta - \vartheta) Q_2(\vartheta)\, d\vartheta,$$

$$\bar{F}_2(\tau) = \int_0^\tau (2\theta - \tau)\varphi_1(\tau - \theta)\varphi_2(\theta)\, d\theta, \quad \varphi_2(\theta) = \int_0^\theta T_2(\theta - \vartheta) Q_1(\vartheta)\, d\vartheta.$$

In the case with measurement data for realizations with sources distributed within circles of radii R_0 and $2R_0$, based on Eq. (5) we may write

$$[1 - \sqrt{s}\sqrt{\lambda c\rho}\, T_1(s)/q_1(s)]^2 = 1 - \sqrt{s}\sqrt{\lambda c\rho}\, T_2(s)/q_2(s),$$

where T_1 and q_1 pertain to the realization with source radius R_0; T_2 and q_2, to the realization with source radius $2R_0$; $q_1(s) = Q_1(s)/(\pi R_0^2)$; and $q_2(s) = Q_2(s)/(4\pi R_0^2)$.

In turn, in the space of inverse transforms we obtain

$$(\lambda c\rho)^{1/2} = \frac{\tilde{\psi}_1(\tau)}{\tilde{\psi}_2(\tau)}, \tag{18}$$

where

$$\tilde{\psi}_1(\tau) = \int_0^\tau \tilde{q}_1(\tau-\theta) \int_0^\theta [2T_1(\vartheta)\tilde{q}_2(\theta-\vartheta) - T_2(\vartheta)\tilde{q}_1(\theta-\vartheta)]\, d\vartheta\, d\theta,$$

$$\tilde{\psi}_2(\tau) = \int_0^\tau T_1(\tau-\theta) \int_0^\theta T_1(\vartheta)\tilde{q}_2(\theta-\vartheta)\, d\vartheta\, d\theta,$$

$$\tilde{q}_1(\theta) = \pi^{-1/2} \int_0^\theta q_1(\vartheta)(\theta-\vartheta)^{-1/2} d\vartheta, \quad \tilde{q}_2(\theta) = \pi^{-1/2} \int_0^\theta q_2(\vartheta)(\theta-\vartheta)^{-1/2} d\vartheta.$$

To determine the parameter a in this case, we can turn to relationship (13) or employ the relation

$$\frac{T_2(s)\, q_1(s)}{T_1(s)\, q_2(s)} = 1 + \exp\left(-\sqrt{\left(\frac{s}{a}\right)} R_0\right),$$

whence

$$[T_2(s)\, q_1(s)]'\, [T_1(s)\, q_2(s)] - [T_2(s)\, q_1(s)]\, [T_1(s)\, q_2(s)]' =$$

$$\frac{R_0}{2\sqrt{as}} [T_1^2(s)\, q_2^2(s) - T_1(s)\, q_2(s)\, T_2(s)\, q_1(s)],$$

which, in the space of inverse transforms, corresponds to

$$a = \frac{R_0^2}{4} \frac{\tilde{\tilde{\psi}}_1^2(\tau)}{\tilde{\tilde{\psi}}_2^2(\tau)}, \tag{19}$$

where

$$\tilde{\tilde{\psi}}_1(\tau) = \pi^{-1/2} \int_0^\tau f_2(\tau-\theta) \int_0^\theta (\theta-\vartheta)^{-1/2} [f_2(\vartheta) - f_1(\vartheta)]\, d\vartheta\, d\theta;$$

$$\tilde{\tilde{\psi}}_2(\tau) = \int_0^\tau (2\theta - \tau)\, f_1(\tau-\theta)\, f_2(\theta)\, d\theta;$$

$$f_1(\theta) = \int_0^\theta q_1(\vartheta)\, T_2(\theta-\vartheta)\, d\vartheta, \quad f_2(\theta) = \int_0^\theta q_2(\vartheta)\, T_1(\theta-\vartheta)\, d\vartheta.$$

The procedure for constructing the above explicit relations for the TPC involves no difficulties owing to a fairly simple representation, in the space of transforms, of the corresponding solutions of the heat conduction problem for definite spatial positions of the points of temperature measurement. By contrast, with an arbitrary disposition of the measurement points on the surface (for example, with heating by a source distributed within a circle), the solution, being an improper integral, is not expressed in terms of some special functions, with account for whose properties it would be possible to construct relatively simple differential relations of the type (9) in the space of transforms.

If we consider a situation where, in one realization with the source dimension R_1, temperatures are measured at the distance R_2 from the source center and, in another with the source dimension R_2, measurements are made at the distance R_1 from the source center, then, based on the relationships

$$T_{R_1}(R_2, s)/q_{R_1}(s) = \frac{R_1}{\lambda} \int_0^\infty \frac{J_0(pR_2)\, J_1(pR_1)}{(p^2 + s/a)^{1/2}}\, dp \equiv \varphi_1(s),$$

$$T_{R_2}(R_1, s)/q_{R_2}(s) = \frac{R_2}{\lambda} \int_0^\infty \frac{J_0(pR_1)\, J_2(pR_2)}{(p^2 + s/a)^{1/2}}\, dp \equiv \varphi_2(s),$$

where $T_{R_1}(R_2, s)$ and $q_{R_1}(s)$ refer to the realization with the source dimension R_1 with the temperature measurement at the point R_2, and $T_{R_2}(R_1, s)$ and $q_{R_2}(s)$ pertain to the realization with the source dimension R_2 with the temperature measurement at the point R_1, it can be shown that it is possible to construct a relatively simple functional relationship between the parameters a and $\lambda c\rho$ and the primary parameters T_{R_1}, T_{R_2}, q_{R_1}, and q_{R_2}. Indeed, using differentiation with respect to the parameter of the Laplace transform s and a series of identity transformations, we write for $\varphi_1(s)$

$$[s\varphi_1'(s)]' + \frac{1}{2}\varphi_1'(s) = \frac{R_1^2 + R_2^2}{4a}\varphi_1(s) + \frac{R_1^2}{2a}\varphi_2(s) - \\ - \frac{R_1^2\sqrt{a}}{4\lambda a\sqrt{s}} - \frac{R_1R_2}{2\lambda a}\int_0^\infty \frac{J_1(pR_1)\, J_1(pR_2)}{(p^2 - s/a)^{1/2}}\, dp, \tag{20}$$

and for $\varphi_2(s)$

$$[s\varphi_2'(s)]' + \frac{1}{2}\varphi_2'(s) = \frac{R_1^2 + R_2^2}{4a}\varphi_2(s) + \frac{R_2^2}{2a}\varphi_1(s) - \\ - \frac{R_2^2\sqrt{a}}{4\lambda a\sqrt{s}} - \frac{R_1R_2}{2\lambda a}\int_0^\infty \frac{J_1(pR_1)\, J_1(pR_2)}{(p^2 + s/a)^{1/2}}\, dp. \tag{21}$$

From Eq. (20) and Eq. (21) it follows that:

$$\left\{ [s\varphi_1'(s)]' + \frac{1}{2}\varphi_1'(s) \right\} - \left\{ [s\varphi_2'(s)]' + \frac{1}{2}\varphi_2'(s) \right\} =$$

$$= \frac{R_1^2 + R_2^2}{4a}[\varphi_1(s) - \varphi_2(s)] + \frac{R_1^2}{2a}\varphi_2(s) - \frac{R_2^2}{2a}\varphi_1(s) - \frac{R_1^2 - R_2^2}{4\lambda a}\frac{\sqrt{a}}{\sqrt{s}}$$

or

$$\frac{4a}{R_1^2 - R_2^2}\left\{ s\,[(\Delta T^{*'} q^* - q^{*'} \Delta T^*)' q^* - 2q^{*'}(\Delta T^{*'} q^* - \Delta T^* q^{*'})] + \right. \\ \left. + \frac{3}{2} q^* (\Delta T^{*'} q^* - \Delta T^* q^{*'}) \right\} = (T_1^* + T_2^*)\, q^{*2} - s^{-1/2}(\lambda c\rho)^{-1/2} q^{*3},$$

$$\Delta T^* = T_1^* - T_2^*, \tag{22}$$

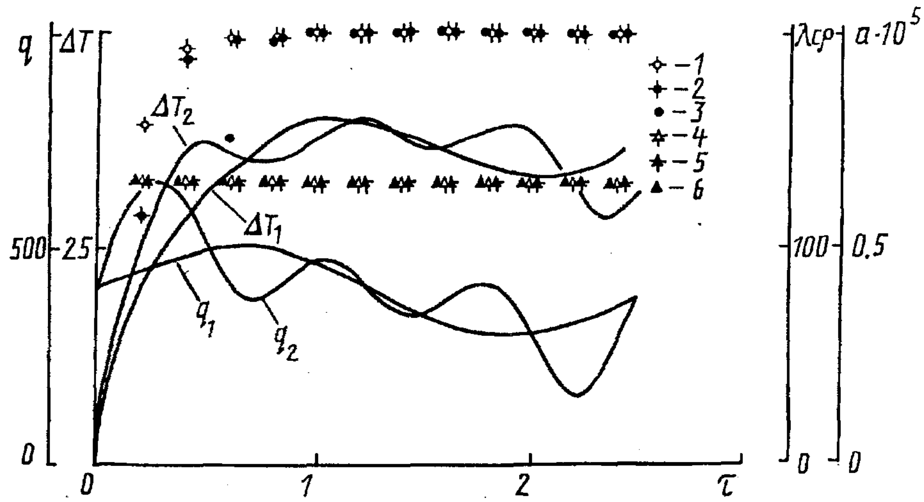

Fig. 1. Initial data for computation and results of evaluating the parameters a and $\lambda c\rho$ for specimen heating by a heat source distributed uniformly within a circle of radius 0.005 m: ΔT and q_1 are the surface temperature of the specimen at the source center and the density of the heat flux from the source in the first realization; ΔT_2 and q_2 are the same, in the second realization; 1, 4) parameters a and $\lambda c\rho$ calculated from ΔT and q of the first realization; 2, 5) the same, calculated from ΔT and q of the second realization; 3, 6) a and $\lambda c\rho$ calculated from the two realizations. ΔT, K; q, kW·m^{-2}; a, m^2·sec^{-1}; $\lambda c\rho$, kW·kJ·m^{-4}·K^{-2}; τ, sec.

where in the case of different laws of time variation of q_1 and q_2: $q^*(s) = q_{R_1}(s)q_{R_2}(s)$, $T_1^*(s) = T_{R_1}(R_2, s)q_{R_2}(s)$, $T_2^*(s) = T_{R_2}(R_1, s)q_{R_1}(s)$ with identical laws of time variation of q_1 and q_2: $q^*(s) = q_{R_1}(s) = q_{R_1}(s)$, $T_1^*(s) = T_{R_1}(R_2, s)$, $T_2^* = T_{R_2}(R_1, s)$. In the space of inverse transforms we thus have the following functional relationship:

$$\frac{4a}{R_1^2 - R_2^2}\Psi_1(\tau) = \Psi_2(\tau) - (\lambda c\rho)^{-1/2}\Psi_3(\tau), \tag{23}$$

where

$$\Psi_1(\tau) = \frac{d\Psi_{1,0}(\tau)}{d\tau} + \Psi_{1,1}(\tau), \quad \Psi_{1,0}(\tau) = \int_0^\tau (3\theta - \tau)\, q^*(\theta) f(\tau - \theta)\, d\theta;$$

$$\Psi_{1,1}(\tau) = \frac{3}{2}\int_0^\tau q^*(\theta) f(\tau - \theta)\, d\theta; \quad f(\theta) = \int_0^\theta (2\vartheta - \theta)\, q^*(\vartheta)\, \Delta T^*(\theta - \vartheta)\, d\vartheta;$$

$$\Psi_2(\tau) = \int_0^\tau \tilde{q}^*(\tau - \theta)\,[T_1^*(\theta) + T_2^*(\theta)]\, d\theta; \quad \tilde{q}^*(\theta) = \int_0^\theta q^*(\vartheta)\, q^*(\theta - \vartheta)\, d\vartheta;$$

$$\Psi_3(\tau) = \pi^{-1/2}\int_0^\tau \tilde{q}^*(\tau - \theta)\int_0^\theta (\theta - \vartheta)^{-1/2} q^*(\vartheta)\, d\vartheta\, d\theta.$$

Based on the relations obtained we developed algorithms for computer calculation of heat transfer constants for various versions of the thermophysical experiment. The algorithms devised were checked by a numerical experiment, where the results of solving the corresponding direct problems of nonsteady heat conduction were taken to be the initial data.

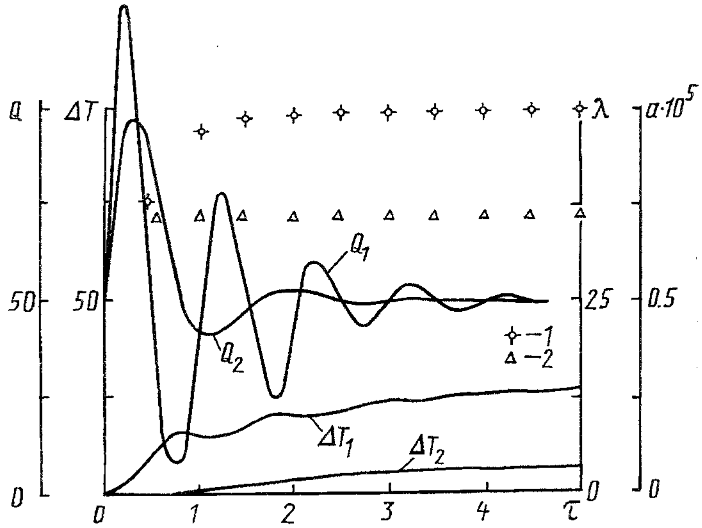

Fig. 2. Initial data for computation and results of evaluating the parameters a and λ for specimen heating by local heat sources concentrated on circles of radii 0.005 and 0.001 m: Q_1 and ΔT_1 are the source power and the temperature at the center of the heating zone in the realization with a 0.005-m source radius; Q_2 and ΔT_2 are the same, in the realization with a 0.01-m source radius; 1, 2) evaluations of the parameters a and λ. Q, W; λ, $W \cdot m^{-1} \cdot k^{-1}$.

Figure 1 gives results of evaluating the parameters a and $\lambda c\rho$ as applied to the version of specimen heating by a local heat source distributed uniformly within a circle of radius 0.005 m, where the following TPC of the material were adopted: $a = 1 \cdot 10^{-5}\ m^2 \cdot sec^{-1}$; $\lambda c\rho = 129.6\ kW \cdot kJ \cdot m^{-4} \cdot K^{-2}$; $\lambda = 0.036\ kW \cdot m^{-1} \cdot K^{-1}$. The evaluations of the parameters a and $\lambda c\rho$, presented in Fig. 1, relate to the case of the "measurement" data for T and q in one realization or in two realizations with different heating conditions. It is clear from the results given in Fig. 1 that the computed values of a and $\lambda c\rho$ converge rather rapidly to their exact values with lengthening of the realization time intervals used for processing.

Figure 2 gives results of evaluating the parameters a and λ using the collection of "measurement" data for T and Q in realizations with sources concentrated on circles of radii 0.005 and 0.01 m. Thermophysical characteristics of the material are taken the same as in the above case. The results of checking the numerical algorithms devised indicate the possibility of effective reconstruction of TPC with the use of the approach proposed.

Similar results also occur for other types of the sources considered in the present work.

Thus, based on the analysis performed it can be concluded that the obtained exact explicit relations connecting TPC of specimens with measurement results for the primary parameters with power of various local heat sources varying arbitrarily in time can be used as a basis for practical methods of identifying TPC of materials.

NOTATION

T, temperature; Q, power of the heat source; q, heat flux; r, z, spatial coordinates; τ, θ, ϑ, time; a, thermal diffusivity; λ, thermal conductivity; c, specific heat; ρ, density.

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